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LETTER TO THE EDITOR

Some generalisations of the O'Raifeartaigh model

Matt Visser†

Physics Department, University of California, Berkeley, California 94720, USA

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Abstract. It is shown that for three or more chiral superfields it is possible to construct an enormous class of non-trivial extensions of the usual three-field O'Raifeartaigh model.

The O'Raifeartaigh model [1] is the simplest example of a theory exhibiting spontaneous breakdown of supersymmetry. The model has found extensive use in phenomenological discussions of supersymmetric grand unified theories where the experimentally required breaking of supersymmetry is typically generated by inclusion of an O'Raifeartaigh sector. It is therefore of some interest to see whether the usual three-field O'Raifeartaigh model has non-trivial extensions.

Such non-trivial extensions do in fact exist for any $n \geqslant 3$ (n = number of chiral superfields). This is shown in two steps. First, we observe that a necessary and *almost* sufficient condition for supersymmetry breaking is that the fermion mass matrix is *everywhere* singular:

$$\det(m(A)) \equiv \det(m_{ab} + g_{abc}A_c) \equiv 0.$$

Second, we shall exhibit a simple construction that leads to a non-trivial everywhere singular mass matrix for any $n \geqslant 3$.

To set the notation, consider a renormalisable n-field Wess–Zumino model [2, 3] described by the superspace Lagrangian:

$$\mathscr{L} = [\bar{\phi}_a \phi_a]_D + 2\,\mathrm{Re}[f(\phi)]_F.$$

The superpotential f is then at most cubic in the superfields ϕ:

$$f(\phi) = \lambda_a \phi_a + \frac{1}{2!} m_{ab}\phi_a\phi_b + \frac{1}{3!} g_{abc}\phi_a\phi_b\phi_c.$$

Following O'Raifeartaigh [1] we define:

$$\lambda_a(A) = \lambda_a + m_{ab}A_b + \frac{1}{2!} g_{abc}A_bA_c$$

$$h_a(A) = m_{ab}A_b + \frac{1}{2!} g_{abc}A_bA_c$$

$$m_{ab}(A) = m_{ab} + g_{abc}A_c.$$

† Present address: Physics Department, University of Southern California, Los Angeles, California 90089-0484, USA.

Both m_{ab} and g_{abc} are completely symmetric in their indices but otherwise arbitrary. In particular they may be complex.

We establish a pair of simple lemmas.

Lemma 1. If supersymmetry is spontaneously broken then $\det(m(A)) \equiv 0$.

Discussion. This result is by now reasonably well known. The first discussion was that of Nicolai [4]. The result has been mentioned and proved in the work of Zumino [5], a rather brief discussion may be found in Cecotti and Girardello [6], and a variation of the result was given by Polchinski [7]. A number of different proofs of this result are possible; we exhibit a version that uses the Witten index [8].

The Witten index $W(\lambda, m, g) = \mathrm{Tr}\{(-1)^F\} = n_B - n_F$ is an integer valued function of the parameters λ, m and g. Now $W(\lambda, m, g)$ is independent of any change of the parameters that does not modify the behaviour of the scalar potential $V(\bar{A}, A) = |\lambda(A)|^2$ as $A \to \infty$. In particular, if $\det(m) \neq 0$ then $W(\lambda, m, g)$ is independent of λ. Further, observe that for any A_0; $W(\lambda, m, g) = W(\lambda(A_0), m(A_0), g)$. This holds because the parameters (λ, m, g) and $(\lambda(A_0), m(A_0), g)$ actually describe the same physical theory. The two apparently different theories are related by a simple change of variables:

$$\phi_{\mathrm{old}}(x, \theta) = \phi_{\mathrm{new}}(x, \theta) + A_0.$$

Suppose now that for some A_0, $\det(m(A_0)) \neq 0$. Then

$$W(\lambda, m, g) = W(\lambda(A_0), m(A_0), g) = W(0, m(A_0), g).$$

But the theory described by $(0, m(A_0), g)$ has a supersymmetry preserving minimum at $A = 0$; further, $\det(m(A_0)) \neq 0$ by construction. Therefore $W(0, m(A_0), g) \geqslant 1$, so $W(\lambda, m, g) \geqslant 1 > 0$. Since the Witten index is non-zero supersymmetry is unbroken [8]. This completes the proof.

The second lemma is trivial.

Lemma 2. If $\det(m(A)) \equiv 0$ then supersymmetry is spontaneously broken for almost all values of λ.

Discussion. Note that $V = |\lambda + h(A)|^2$. Thus $V(A, \bar{A}) = 0$ if and only if $-\lambda \in \mathrm{image}[h]$. But

$$\begin{aligned} \mathrm{Volume}(\mathrm{image}[h]) &= \int \mathrm{d}^n A \, \mathrm{d}^n \bar{A} |\partial h / \partial A|^2 \\ &= \int \mathrm{d}^n A \, \mathrm{d}^n \bar{A} |\det(m(A))|^2 \\ &= 0. \end{aligned}$$

Thus image$[h]$ is a set of measure zero in $\mathbb{C}^n$ and the theory described by (λ, m, g) almost always spontaneously breaks supersymmetry.

Rephrasing this result: a necessary and almost sufficient condition for the spontaneous breaking of supersymmetry in a Wess–Zumino model is that $\det(m(A)) \equiv 0$. The search for generalisations of the O'Raifeartaigh model is thus reduced to the task of finding symmetric tensors that satisfy

$$\det(m_{ab} + g_{abc} A_c) \equiv 0.$$

We wish to find symmetric tensors satisfying

$$\det(m_{ab}+g_{abc}A_c)\equiv 0.$$

Such tensors may easily be generated by first considering the set of 'leading triangular' matrices. A matrix will be said to be leading triangular if all its elements below the principal transverse are equal to zero:

$$X=\begin{pmatrix} X_{11} & X_{12} & & \cdots & & X_{1n} \\ X_{21} & & & & X_{2;n-1} & 0 \\ \vdots & & \cdot^{\cdot^{\cdot}} & & 0 & \vdots \\ & & & \cdot^{\cdot^{\cdot}} & & \\ \vdots & X_{n-1;2} & 0 & & & \\ X_{n1} & 0 & & \cdots & & 0 \end{pmatrix}.$$

For any leading triangular matrix the determinant is simply the product of the elements along the principal transverse:

$$(\det X)=X_{1;n}X_{2;n-1}X_{3;n-2}\cdots X_{n;1}.$$

Singular leading triangular matrices are thus easily fashioned by setting one of the elements along the principal transverse equal to zero. Then any linear combination of singular leading triangular symmetric matrices will itself be singular leading triangular and symmetric provided that the matrices all have a common zero along the principal transverse.

Generalised O'Raifeartaigh models may now be constructed as follows.

(a) Let the $n+1$ matrices m_{ab}, $(g_{ab})^c$ be leading triangular, symmetric and singular with a common zero along the principal transverse.

(b) Impose the symmetry condition

$$(g_{ab})^c=g_{abc}=g_{(abc)}.$$

(c) The model described by the parameters $(\lambda_a, m_{ab}, g_{abc})$ has a spontaneously broken supersymmetry for almost all values of λ_a.

For a specific example, consider the $n=3$ case and choose

$$m_{ab}=\begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & 0 & 0 \\ M_{13} & 0 & 0 \end{pmatrix}$$

$$(g_{ab})^c=\left(\begin{array}{ccc|ccc|ccc} \alpha & \beta & \gamma & \beta & 0 & 0 & \gamma & 0 & 0 \\ \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

Observe that this is already a generalisation of the usual O'Raifeartaigh model. A comparison with O'Raifeartaigh's paper [1] shows that the usual model is recovered by setting $M_{13}=\alpha=\beta=0$ and redefining $M_{11}=\mu$; $M_{12}=M$; $\gamma=g$.

As a second and final example consider the $n=4$ case. We can choose

$$m_{ab}=\begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{12} & M_{22} & 0 & 0 \\ M_{13} & 0 & 0 & 0 \\ M_{15} & 0 & 0 & 0 \end{pmatrix}$$

$$(g_{ab})^c = \left(\begin{array}{cccc|cccc|cccc|cccc} \alpha & \beta & \gamma & \delta & \beta & \varepsilon & 0 & 0 & \gamma & 0 & 0 & 0 & \delta & 0 & 0 & 0 \\ \beta & \varepsilon & 0 & 0 & \varepsilon & \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

This procedure may be used for any arbitrary n, thereby generating an enormous class of generalised O'Raifeartaigh models.

It is instructive to point out what has not been done. First, no claim is made that this is the most general construction for O'Raifeartaigh models. More general construction may in principle be possible. However, for the special case $n=3$ one may, by exhaustive consideration of the possibilities, convince oneself that the $n=3$ model described above is indeed the most general non-trivial model to within non-singular transformations of the type

$$m_{ab} \to V_{a\bar{a}} V_{b\bar{b}} m_{\bar{a}\bar{b}}$$

$$g_{abc} \to V_{a\bar{a}} V_{b\bar{b}} V_{c\bar{c}} g_{\bar{a}\bar{b}\bar{c}}.$$

Second, note that we have never had to calculate the location of any absolute minimum, nor the value of $V(A, \bar{A})$ at that minimum.

There is an enormous number of non-trivial O'Raifeartaigh models for three or more chiral superfields. It is relatively easy to check whether or not any particular models entertains the possibility of spontaneous breakdown of supersymmetry. However actually finding the location and value of the absolute minimum of $V(A, \bar{A})$ is in general exceedingly difficult.

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